+3 SEM-1 (NEP)

MAT-C1(P-2) (ARTS/SC)

DECEMBER, 2024

Introduction to Algebra & Number Theory

Full marks-100 Time-3hrs

Figures in the right-hand margin indicates marks

GROUP-A

1. Answer **all** questions. $[1\times10]$
   a. What is the order of the element 3 in Z_8 ?
   b. The group $(Z,+)$ is cyclic . (**TRUE/ FALSE**)
   c. Find the g.c.d of 528 and 231?
   d. Define a subring?
   e. Give an example of a finite field?
   f. Write Wilson's theorem.
   g. Number of generators of the group Z_{20} is _____.
   h. Write the first three Fermat numbers.
   i. Every subgroup of an abelian group is a normal subgroup. (**TRUE/ FALSE**)
   j. Define characteristics of a ring.

GROUP-B

2. Answer **all** questions. $[2\times9]$
   a. If $a\equiv b(\mathrm{mod}\,n)$ then Prove that $b\equiv a(\mathrm{mod}\,n)$.
   b. Show that inverse of every element is unique in a group.
   c. Find the subgroup $\langle 4\rangle$ in Z_{10}.
   d. Let A= {1,2,3}. Give an example of a symmetric relation and a transitive relation on A.
   e. Let $a,b,c\in Z$. If 'a divides b' and 'a divides c' , then prove that a divides $b+c$.
   f. Define an integral domain. Is the ring of integers an integral domain?
   g. State Fermat's little theorem.
   h. Find g.c.d of 40 and 96 by using Euclidean algorithm.
   i. Show that $2Z\cup 3Z$ is not a subring.

GROUP-C

3. Answer any **eight** questions. $[5\times8]$
   a. Show that every cyclic group is abelian but the converse is not true.
   b. Define Center of a group G. Show that center of a group is a normal subgroup of G.
   c. Prove that the relation " congruence modulo n" is an equivalence relation on the set of integers.
   d. Define a field. Prove that a finite integral domain is a field.
   e. Define Euler's Φ - function. Find $\Phi(100),\Phi(63),\Phi(40)$.
   f. Use mathematical induction to prove that $n^3+(n+1)^3+(n+2)^3$ is divisible by 3.
   g. Write all elements of D_4. Find the order of each element in that group. Also find $Z(D_4)$.
   h. Define a prime number. Show that there are infinitely many primes.
   i. If a and b are relatively prime integers then prove that gcd $(a+b,a-b)$ =1 or 2.
   j. Determine whether the integer 701 is prime by testing all primes $p\le\sqrt{701}$.

GROUP-D

4. Answer any **four** questions. $[8 \times 4]$

a. State and prove fundamental theorem of Arithmetic.

b. When a linear Diophantine equation $ax + by = c$ is solvable. Find the general solution of linear Diophantine equation $63x - 23y = -7$.

c. Define a normal subgroup. Show that intersection of two normal subgroups of G is a normal subgroup of G.

d. Solve the following system of congruences by using Chinese remainder theorem.

$$x \equiv 1 (\mathrm{mod}\, 3)$$
$$x \equiv 2 (\mathrm{mod}\, 5)$$
$$x \equiv 3 (\mathrm{mod}\, 7)$$

e. State and prove fundamental theorem for cyclic groups.

THE END

DECEMBER,2024
CALCULUS & ANALYTIC GEOMETRY

Full marks-100 Time-3 hrs

Symbols has usual meanings
Figures in the right-hand side of margin indicates marks

GROUP-A

1. Answer **all** questions. $[1\times10]$

   a. Find the derivative of $\cosh(\log 2x)$.

   b. How many loops are there in the curve $r = a\sin 3\theta$?

   c. Find the asymptotes of the curve $x^3 + y^3 = 3axy$.

   d. The curve $f(x) = e^{2x}$ is always concave upward .**(TRUE / FALSE)**

   e. Define an inflection point of a curve.

   f. Give an example of the indeterminate form 0^0 .

   g. Evaluate $\int_0^{\pi/2} cos^5 x\, dx$.

   h. The value of $\vec{a}.\left(3\vec{b}\times 4\vec{a}\right)$ is _____.

   i. Write the parametric form of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

   j. If $r(t) = 3\cos t\,\hat{i} + 2\sin t\,\hat{j}$, then find the velocity at $t = 0$.

GROUP-B

2. Answer **all** questions. $[2\times9]$

   a. If $\cosh x = \frac{6}{5}$ then find the values of $\sinh x$ and $\tanh x$.

   b. Evaluate $\int_0^{\ln 3} \frac{e^x - e^{-x}}{e^x + e^{-x}}\, dx$.

   c. Find the third derivative of $y = \ln(2x+3)$.

   d. Find the interval in which the function $f(x) = \frac{x}{x^2+1}$ is increasing.

   e. Find the vertical and horizontal asymptote of the curve $f(x) = \frac{3+2x}{7x-8}$

   f. Evaluate $\lim_{x\to 0} \frac{e^{3x} - 3x - 1}{1-\cos x}$.

   g. Find the volume of the solid when the area bounded by the curve $y = \sqrt{x}$, the x-axis and the line $x = 2$ revolved about the x-axis.

h. Find the equation of the tangent line to the curve $x = 2t + 4,\ y = 8t^2 - 2t + 4$ at $t = 2$ without eliminating the parameter.

i. Are the vectors $\vec{a} = \hat{i} + 3\hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} - \hat{j} - \hat{k}$, $\vec{c} = 7\hat{j} + 3\hat{k}$ coplanar?

GROUP-C

3. Answer any **Eight** questions. $[5 \times 8]$

a. Trace the curve $r = a(1 + \cos\theta)$.

b. Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{n}{(n+k)^2}$ using Riemann Sum.

c. Examine the concavity and inflection points of the curve $f(x) = x^3 - 3x^2 + 2x$.

d. Derive the reduction formula for $\int \sec^n x\, dx$.

e. State the Leibnitz's theorem. Using Leibnitz theorem to find y_4 if $y = x^4 \cos x$.

f. Find the arc length of the curve $24xy = y^4 + 48$ from $y = 2$ to $y = 4$.

g. Using Washer method find the volume of the solid generated when the region enclosed by the curves $x = \sqrt{y}$ and $x = \frac{y}{4}$ is revolved about $x-$axis.

h. Sketch the hyperbola and find the vertices, foci and asymptotes for the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$.

i. Find the unit tangent and unit normal vector to the curve n

j. When two vectors are orthogonal? If a & b are orthogonal unit vectors then find $(a \times b) \times a$ and $(a \times b) \times b$.

GROUP-D

4. Answer any **four** questions. $[8 \times 4]$

a. If $y = \cosh(\sin^{-1} x)$, then show that $(1 - x^2)y_{n+2} - (2n+1)x\, y_{n+1} - (n^2 + 1)y_n = 0$.

b. Find the asymptotes of the curve $x^3 + 4x^2y + 4xy^2 + 5x^2 + 15xy + 10y^2 - 2y + 1 = 0$.

c. Derive Walli's reduction formula for $\int_0^{\pi/2} sin^n x\, dx$. Use it to find $\int_0^{\pi/2} sin^7 x\, dx$.

d. Classify the conic given by $4x^2 - 4xy + y^2 - 8x - 6y + 5 = 0$. Find its vertex, focus and directrix.

e. The position vector of an object moving in space is given by $r(t) = e^{-t}\cos t\,\hat{i} + e^{-t}\sin t\,\hat{j} + e^{-t}\,\hat{k}$. Find the velocity, acceleration of the object at arbitrary time t , also Find the curvature of the trajectory at time t.

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CALCULUS & ANALYTIC GEOMETRY

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GROUP-A

1. Answer **all** questions. $[1\times10]$

   a. The value of $\cosh^2 x-\sinh^2 x=$_____.

   b. Find the derivative of $\sinh\left(x^3\right)$.

   c. How many indeterminate forms are there?

   d. A curve $y=f(x)$ is concave upward if $\frac{d^2y}{dx^2}$ is _____.

   e. Find the asymptote parallel to co-ordinate axes for the curve $x^2y=2+y$.

   f. Evaluate $\int_0^1 \frac{x}{x+1}\,dx$.

   g. $a\times(b\times c)=(a\times b)\times c$. **(TRUE / FALSE)**

   h. Let $F(t)=2t\,\hat{i}-5\hat{j}+t^2\hat{k}$ and $G(t)=(1-t)\hat{i}+\frac{1}{t}\hat{k}$, then find the dot product $F(t).G(t)$

   i. Evaluate $\int\left\langle e^{2t},e^{-t},t^2\right\rangle dt$.

   j. Give two examples of central conics.

GROUP-B

2. Answer **all** questions. $[2\times9]$

   a. Find the value of $\tanh(\ln 4)$.

   b. If $y=\log\sin(2x)$ then find y_2.

   c. Show that the function $y=\log x$ is always concave downward.

   d. Find the vertical and horizontal asymptote of the curve
   $$y=\frac{x^2+3x-5}{x^2-5x+6}.$$

   e. Evaluate $\lim_{x\to\pi}(x-\pi)\,cotx$.

   f. Evaluate $\int_0^{\pi/2} sin^4x\,dx$.

   g. Find the volume of the solid when the area bounded by the curve $y=\sqrt{\cos x}$ and the x-axis from $\frac{\pi}{4}$ to $\frac{\pi}{2}$ is revolved about x- axis.

   h. Find $u\times v$ when $u=3\hat{i}+\hat{k}$ and $v=2\hat{i}-7\hat{j}$.

i. Prove that $(a\times b)\times c+(b\times c)\times a+(c\times a)\times b=0$.

GROUP-C

3. Answer any **Eight** questions. $[5\times 8]$

a. If $x=\sin t, y=\sin pt$, then prove that $(1-x^2)y_2-xy_1+p^2y=0$.

b. Determine the concavity and find the inflection points of the curve $f(x)=3x^4-4x^2+1$.

c. Derive the reduction formula for $\int(\log x)^n\,dx$. Use it to evaluate the integral $\int(\log x)^4\,dx$.

d. Evaluate $\int\frac{1+x}{(2+x)^2}e^x\,dx$.

e. Use cylindrical shells to find the volume generated when the area bounded by the curves $y=1-x^2\ , y=0, x=0$ is revolved around the x-axis.

f. Trace the curve $x^3y=x^2-1$.

g. What is reduction formula. Obtain the reduction formula for the integral $I_n=\int x^n e^x\,dx$.

h. Find the length of the curve $y=\frac{x^3}{24}+\frac{2}{x}$ from $x=2$ to $x=3$.

i. Find the tangent vector to the graph of the given vector function F, defined by $F(t)=t^2\hat{i}+(\cos t)\hat{j}+(t^2\cos t)\hat{k}$ at t=0 , $\frac{\pi}{2}$.

j. Find the constant **'a'**, so that the following vectors lie in the same plane.
$u=2\hat{i}-\hat{j}+\hat{k}$, $v=\hat{i}+2\hat{j}-3\hat{k}$, $w=3\hat{i}+a\,\hat{j}+5\hat{k}$.

GROUP-D

4. Answer any **Four** questions $[8\times 4]$

a. If $y=\sin\left(m\sin^{-1}x\right)$, then show that
$(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}+(m^2-n^2)y_n=0$.

b. Find the concavity and inflection points of the curve $f(x)=x^4-6x^3+12x^2+5x+7$.

c. Define an asymptote of a curve. Find the asymptotes of the curve
$f(x)=2x^3-x^2y-2xy^2+y^3-4x^2+8xy+4x$.

d. Find the area of the surface generated by revolving the curve $8xy^2=2y^6+1\ \ , 1\le y\le 2$ about y-axis.

e. Calculate the unit tangent vector $T(t)$, unit normal vector $N(t)$ and unit binormal vector $B(t)$ of the vector valued function $r(t)=\cos t\,\hat{i}+\sin t\,\hat{j}+\hat{k}$ at t = $\frac{\pi}{4}$.

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