+3 Sem-I(NEP)
PHY-C-1(P-I)

December, 2024

Mathematical Physics-I

Full Mark-100 Time-3 hours

Answer any **ALL** questions.

The figures in the right-hand margin indicate marks.
Answers to all bits should be done at one place.

1. Answer the following questions [1 x 10]

a. The value of triple product $\vec{\mathbf{A}}.(\vec{\mathbf{A}}\ \mathbf{X}\ \vec{\mathbf{B}})$ is _____.

b. Curl $\vec{\mathbf{r}}$ =_____. ($\vec{r}$ is the position vector)

c. Define Stoke's theorem.

d. If $\vec{V} = 2x\hat{i} + y\hat{j} + z\hat{k}$, find div $\vec{V}$.

e. Find the value of 'n' for which the vector $r^n\boldsymbol{r}$ is solenoidal, where $\boldsymbol{r}$ is the position vector.

f. Define Dirac delta function.

g. Write the scalar factor of spherical coordinates.

h. State two properties of delta function.

i. The Wronskian of the function $y_1 = sinx$ and $y_2 = sinx - cosx$ is _____ .

j. Find the integrating factor of the equation $sec^2y\ \frac{dy}{dx} + x\tan y = x^3$.

2. Answer all questions in maximum 50 words. [2 x 9]

a. Write the transformation equation from rectangular coordinates to spherical coordinate system.

b. Show that the cylindrical coordinate system is orthogonal.

c. Find the particular integral of the equation $y'' - 3y' + 2y = e^x$.

d. Find the volume of the parallelepiped whose coterminous edges are given by the vectors: $\hat{i} - \hat{j} + 3\hat{k}$; $\hat{i} + \hat{j} - 2\hat{k}$ and $2\hat{i} + \hat{j} + 4\hat{k}$.

e. If for three vectors a.(b × c) = 6 and |a| = 2, |b| = 3, |c| = 1, find the angle θ between b and c.

f. Find grad(*f*) if *f*(x,y,z) = xy + y^2z at the point (0, 1, -1).

g. Evaluate the integration $\iint_S \vec{F}.\hat{n}\,dS$, where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$ and $\vec{F} = 3x\hat{i} + 4y\hat{j} + 5z\hat{k}$.

h. If *f*(x, y) = $4x^3 - 3x^2y^2 + 2x + 3y$, find the partial derivatives *fx* and *fy*.

i. Evaluate $\int_2^6 (3x^2 - 2x - 1)\delta(x - 3)dx$.

3. Answer any 8 questions in maximum 250 words. [5 x 8]

a. If $\vec{F} = xz\hat{i} + xy\hat{j} + (x^2 + y^2)\hat{k}$, the find the Div $\vec{F}$ in terms of cylindrical coordinates.

b. The point with spherical coordinates are given as (8, 2π/3, π/3). What is the value of (x, y, z) in Cartesian coordinate system?

c. Show that $x\frac{d}{dx}(\delta(x)) = -\delta(x)$.

d. Given u = yz/x, v = zx/y, w = xy/z, then find the value of $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

e. Determine the net flux of the vector field $\vec{F} = 2x^2y\hat{i} + z\hat{j} + y\hat{k}$ emerging from the unit cube $0 \leq x, y, z \leq 1$.

f. Find the general solution to the equation: $4y'' - 4y' + y = 0$.

g. Find the solution to the equation: $Cos^2x\frac{dy}{dx} + y = \tan x, (0 \leq x \leq \frac{\pi}{2})$.

h. Find the solution to the differential equation: $(x + y)^2\frac{dy}{dx} = a^2$.

i. Calculate the line integral $\oint_C x^2y\,dx + (y - 3)dy$, where C is a rectangle with vertices (1,1), (4,1), (4,5) and (1,5).

j. Maximize $u = 4x^2 + 3xy + 6y^2$ subject to x + y =56.

4. Answer any 4 questions . [8 x 4]

a. Solve the initial value problem: $y'' - 2y' + y = \sin x, y(0) = -2, y'(0) = 2$.

b. Verify divergence theorem for the vector $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$ taken over the cube $0 \leq x, y, z \leq 2$.

c. Show that the gradient of a real-valued function F(ρ,θ,φ) in spherical coordinates is $\nabla F = \frac{\partial F}{\partial \rho}e_\rho + \frac{1}{\rho\, Sin\,\varphi}\frac{\partial F}{\partial \theta}e_\theta + \frac{1}{\rho}\frac{\partial F}{\partial \varphi}e_\varphi$.

d. Verify Stoke' theorem for the vector $\vec{A} = (2x - y)\,\hat{i} - yZ^2\,\hat{j} + y^2z\,\hat{k}$A over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$.

e. i. Show that the vectors i + 2j − 3k, 2i − j + 2k and 3i + j − k are coplanar.

ii. Let $f(x, y, z) = xye^{x^2+z^2-5}$. Calculate the gradient of f at the point (1,3,−2) and calculate the directional derivative at the point (1,3,−2) in the direction of the vector **v** = (3,−1,4).

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DECEMBER, 2024
(Mechanics)

Full Marks: **100** Time: **3 hours**

*Answer **all** questions*

The figures in the right hand margin indicates marks

1. Answer all the following objectives **[10 x 1]**

i. The moment of inertia of earth I. What will be the moment of inertia if The mass is doubled.

(a) $\frac{I}{4}$ (b) I

(c) 2I (d) $\frac{I}{2}$

ii. State Routh rule.

iii. The relationship among Young's modulus (Y) , Rigidity modulus (G) and Poisson ratio(μ) is

(a) $G = \frac{Y}{2(1+\mu)}$ (b) $G = \frac{Y}{2(1-\mu)}$

(c) $G = \frac{Y-1}{2(1+\mu)}$ (d) $G = \frac{Y}{1+\mu}$

iv. Write down the expression for twisting torque of a solid cylinder.

v. The bending moment produced in beam is _____ to the moment of inertia.

(a) directly proportional (b) Inverse proportional

(c) Linear (d) None

vi. A particle of mass m moves under the action of a central force whose potential is $V(r) = kmr^3$ (K > 0), then the angular frequency is

(a) $\sqrt{3ka}$ (b) $\sqrt{ka}$

(c) $\sqrt{5ka}$ (d) $\sqrt{15ka}$

vii. Distinguish between gravitational mass and inertial mass.

viii. What will be the impacts on Einstein's special theory of relativity if the velocity of light is not 3×10^8 m/sec.?

ix. The time interval between two events in rest frame is Δt. If it is measured from a moving frame, it is Δt^I, then

(a) $\Delta t = \Delta t^I$ (b) $\Delta t^I < \Delta t$

(c) $\Delta t^I > \Delta t$ (d) $\Delta t^I = \sqrt{2}\,\Delta t$

x. Write down the mass-energy equivalence in relativity.

2. Very short answer (Answer all) **[9 x 2]**

i. Define the moment of inertia of a body and discuss its physical significance.

ii. Calculate the moment of inertia of a hollow cylinder of mass M and radii R_1 and R_2 about its symmetry axis.

iii. State perpendicular axis theorem.

iv. Write down the expression for power dissipation by the oscillator under forced vibration.

v. What is gravitational potential and gravitational potential energy ? Write down their expression.

vi. What is meant by central force ? How it is different from centrifugal and coriolis force.

vii. A spaceship of (rest) length 100 m takes 5 μs to pass an observer on earth. What is its speed relative to earth.

viii. A motorist passes through a red light signal and when challenged he claims that the signal colour he actually saw was green ($\lambda = 5.4 \times 10^{-7}$ m), not red ($\lambda = 6.2 \times 10^{-7}$ m) due to Doppler effect. What is his speed ?

ix. A circular lamina moves with its plane parallel to the X-Y plane of rest frame S. Assuming its motion to be along X-axis, calculate the velocity at which its surface area would appear to be reduced to half to an observer.

3. Short Answer (Answer 8 out of 10 questions) [8 x 5]

i. Obtain the moment of inertia of flywheel about its axis of rotation.

ii. Obtain Euler's equation of motion of a rigid body.

iii. Write a brief note on the sharpness of resonance

iv. Obtain an expression for Quality factor in forced oscillation.

v. Derive the expression for the bending moment of beam.

vi. Explain the concept of power law potential in brief..

vii. Discuss how a two body central force problem can be reduced to an equivalent one body problem.

viii. Explain the superiority of Lorentz transformation over Gallelian transformation.

ix. In a certain inertial frame two light pulses are emitted at points 5 k.m apart and separated in time by 5 μs. An observer moving at a speed v along the line joining these points. Considering the pulses to be simultaneous, calculate the value of v.

x. Write down the expression for

(a) relativistic mass m and discuss the cases where (i) $v << c$ (ii) $v \rightarrow c$. Also plot $\frac{m}{m_0}$ vs $\frac{v}{c}$

(b) relativistic momentum p and discuss the cases where (i) $v << c$ (ii) $v \rightarrow c$. Also plot p vs $\frac{v}{c}$

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4. Long Answer (Answer 4 out of 5 questions) [4 x 8]

i. Get an expression for kinetic energy of a rigid body. Hence, obtain the same if the rigid body undergoes both rotation and translation motion.

ii. Set up the differential equation of Damped Harmonic Oscillator and solve it for critical damping and under damping cases.

iii. Derive the expressions for gravitational potential and field due to a thin spherical shell at a point (a) outside (b) inside and (c) on the surface of the shell.

iv. Derive Lorentz space-time transformation formulae in detail.

v. Derive energy-momentum relation in relativity.

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+3 SEM−1 (NEP)

PHY−C−2 (P−1)

DECEMBER, 2024

(Mechanics)

Full Marks: **100** Time: **3 hours**

*Answer **all** questions*

The figures in the right hand margin indicates marks

1. Answer the following **[10 x 1]**

a) The moment of inertia of a solid sphere about its tangent is-

(a) $\frac{7}{5}MR^2$ (b) $\frac{1}{5}MR^2$

(c) $\frac{1}{2}MR^2$ (d) $\frac{7}{5}MR$

b) Which one of the following particles experiences a Coriolis force?

(a) A particle at rest w.r.t. earth at Bhopal

(b) A particle thrown vertically upward at Bhopal.

(c) A particle thrown vertically upward at the north pole.

(d) A particle moving horizontally along the north-south direction at Bhopal.

c) Soap bubble is spherical due to

(a) viscosity (b) elasticity

(c) surface tension (d) none

d) In forced oscillation, the resonance frequency for which the amplitude of the forced harmonic oscillator becomes maximum, is given by -

(a) $\frac{P_r}{2\pi}=\frac{\sqrt{\omega^2+b^2}}{2\pi}$ (b) $\frac{P_r}{2\pi}=\frac{\omega-2b}{2\pi}$

(c) $\frac{P_r}{2\pi}=\frac{\sqrt{\omega+b^2}}{2\pi}$ (d) $\frac{P_r}{2\pi}=\frac{\sqrt{\omega^2-2b^2}}{2\pi}$

e) The dimension of quality factor in damped harmonic oscillator is _____.

f) Distinguish between gravitational mass and inertial mass.

g) A particle moves in inverse square law central force. If the ratio of maximum angular velocity to minimum angular velocity is n, find eccentricity.

h) What is meant by time dilation.

i) Although mass-energy equivalence of special relativity allows conversion of a photon to an electron-positron pair such a process cannot occur in free space because

(a) The mass is not conserved.

(b) The energy is not conserved.

(c) The momentum is not conserved.

(d) The charge is not conserved.

2. Very short answer (Answer all) **[9 x 2]**

a) Distinguish between centrifugal force and Coriolis force with examples and write their expression.

b) Calculate the moment of inertia of a solid uniform sphere of mass M and radius0 R about a diameter.

c) State perpendicular axis theorem.

d) What do you understand by the quality factor (Q) of a damped harmonic oscillator?

e) State Newton's law of gravitation and mention its dimension.

f) What is meant by weightlessness in satellite.

g) State the postulates of Einstein's special theory of relativity.

h) A certain material has density δ_0 when it is at rest. What is the apparent density when its velocity is v ?

i) Calculate the percentage of contraction in the length of a rod in a frame of reference moving with velocity 0.8C in a direction parallel to its length.

3. Short Answer (Answer 8 out of 10 questions) [8 x 5]

a) State and prove parallel axis theorem.

b) Calculate the moment of inertia of a spherical shell of mass M and radii R_1 and R_2 about a tangent.

c) Write a note on velocity resonance.

d) Derive the expression for the bending moment of beam.

e) Establish the relation between elastic consants.

f) Write down the kepler's laws of planetary motion.

g) A particle of mass m moves under the action of a central force whose potential is $V(r) = kmr^3$ ($K > 0$). Find its angular frequency.

h) Write and explain Lorentz transformation equations in detail.

i) A car of rest length 5 m passes through a garage of length 4 m. Due to relativistic contraction the car is only 3 m long in the moving frame. Find the relativistic speed of the car.

j) A lump of clay has a rest mass 4 kg. While travelling with speed of light $\frac{3}{5}c$ it collides head on with an identical lump going in opposite direction with same speed. If the lumps stick together what will be the mass of composite ?

4. Long Answer (Answer 4 out of 5 questions) [4 x 8]

a) Get an expression for kinetic energy of a rigid body. Hence, obtain the same if the rigid body undergoes both rotation and translation motion.

b) Obtain Poiseuill's equation for flow of a liquid and discuss its corrections.

c) Discuss and write down the differential equation of motion with central force and its solution

d) Set up the differential equation of Damped Harmonic Oscillator and solve it for critical damping and under damping cases.

e) Derive energy-momentum relation in relativity.

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