

April,2026
Real Analysis -I

Full Marks: 100

Time: 3 hours

Answer as per instruction. Symbols have their usual meanings
Figures in the right-hand margin indicates marks

NO.1 Answer all the Question.

10x1 =10

- a) If x is Irrational then x^2 is Irrational (True/False)
- b) Let $S = \left\{1 - \frac{(-1)^n}{n}, n \in \mathbb{N}\right\}$ then find $\inf S$.
- c) The set of Irrational number is countable.(True/False)
- d) Find the limit of the sequence $a_n = n^{\frac{1}{n}}$.
- e) Write the necessary condition for the convergence of the series $\sum a_n$
- f) Find $\lim_{n \rightarrow \infty} \sqrt{n+1} - \sqrt{n}$
- g) A function $f: \mathbb{R} \rightarrow \mathbb{R}$ is defined by $f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ 1-x & \text{if } x \notin \mathbb{Q} \end{cases}$, the function $f(x)$ is continuous at $x = \underline{\hspace{2cm}}$
- h) Give an example of function which is continuous but not uniformly continuous.
- i) Give an example of a series which is conditionally convergent.
- j) Define Cauchy Sequence in $\mathbb{R}$.

No. 2 Answer **All** the Question.

9x2=18

- a) State supremum property of Real Number.
- b) If $a < b$ and $c < d$ then prove that $a + c < b + d$
- c) Give two example of bounded sequence which is not convergent.
- d) Show that $\lim_{x \rightarrow 0} \sin \frac{1}{x}$ does not exist.
- e) State integral test for convergence of a series.
- f) Give an example of a function $f: \mathbb{R} \rightarrow \mathbb{R}$ which is continuous only at one point.
- g) Write the behaviour of the series $\sum \frac{1}{n^p}$ for different values of p .
- h) Show that $\lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$
- i) Show that the sequence $a_n = (-1)^{n-1}$ is not convergent.

No.3 Answer **Any Eight** Question.

8x5 = 40

- a) State Density property of rational number, Prove that in between any two real numbers there is a rational number.
- b) Prove the interval $(0,1)$ is uncountable.
- c) Show that Every Convergent sequence is a Cauchy's Sequence.

- d) Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q} \\ -1 & \text{if } x \notin \mathbb{Q} \end{cases}$, Show that $\lim_{x \rightarrow c} f(x)$ does not exist for any $c \in \mathbb{R}$
- e) Show that the series $\sum \frac{1}{n}$ is not convergent.
- f) Show the every absolute convergent series is convergent.
- g) Show that a Sequence can have at most one Limit.
- h) Let $D \subset \mathbb{R}$, f, g be function on D to $\mathbb{R}$ if $\lim_{x \rightarrow c} f(x) = L_1$ and $\lim_{x \rightarrow c} g(x) = L_2$ then show that $\lim_{x \rightarrow c} f + g(x) = L_1 + L_2$
- i) Show that Every Bounded sequence of real number has a convergent subsequence.(Bolzano Weierstrass Theorem for sequence)
- j) Show that the function f defined by $f(x) = \frac{1}{x}$ $x \in [1, \infty)$ is uniformly continuous on $[1, \infty)$

No. 4 Answer **any Four** Question.

4 X 8 =32

- a) State and Prove Nested Interval Property.
- b) Let $X = a_n$ be a decreasing sequence of strict positive number with $\lim a_n = 0$ then show that the alternating series $a_1 - a_2 + a_3 - a_4 + \dots$ is convergent.
- c) Let $I = [a, b]$ and let $f: I \rightarrow \mathbb{R}$ be continuous on I , If $f(a)$ and $f(b)$ are of opposite sign then prove that there exist at least one $c \in I$ such that $f(c) = 0$ (Location of Root Theorem)
- d) Prove that a continuous function on a closed and bounded interval I is uniformly Continuous (Uniform Continuity Theorem)
- e) Test the convergence of the Series (Any Two)

i. $\frac{1+2}{2^3} + \frac{1+2+3}{3^3} + \frac{1+2+3+4}{4^3} + \dots + \dots$

ii. $1 + \frac{1}{2} \frac{1}{3} + \frac{1.3}{2.4} \frac{1}{5} + \frac{1.3.5}{2.4.6} \frac{1}{7} + \dots + \dots$

iii. $1 - \frac{2^2}{2!} + \frac{3^3}{3!} - \frac{4^4}{4!} + \dots$

April,2026
Algebra-I

Time: 3 hours

Full marks:100

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1. Answer **ALL** the followings.

[10x1 = 10]

- i) The Order of permutation group $S_5 = \underline{\hspace{2cm}}$.
- ii) Let H be subgroup of G . For $a, b \in G$, $|aH| = |bH|$. **(True/False)**
- iii) Let $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 1 & 2 & 4 & 3 & 5 \end{pmatrix}$, write the permutations α in cycle notation.
- iv) Find the order of $(2,3)$ in $\mathbb{Z}_4 \oplus \mathbb{Z}_6$.
- v) Find a normal subgroup of D_4 . (D_4 is the dihedral group of order 8.)
- vi) What is the characteristic of the ring $\mathbb{Z}_{10}$.
- vii) Find all the prime ideals of $\mathbb{Q}(+, \times)$.
- viii) Is the map $\phi: \mathbb{Z}(+, \cdot) \rightarrow \mathbb{Z}(+, \cdot)$ defined by $\phi(x) = 2x$ a ring homomorphism? **(YES/NO)**
- ix) $f(x) = 2x^2 + 4$ is irreducible over $\mathbb{Z}$. **(True/False)**
- x) Give an example of a Euclidean domain.

2. Answer **ALL** the followings.

[9 x 2 = 18]

- i) State Cayley's Theorem.
- ii) Let H be a subgroup of G , and let $a \in G$. Then show that aH is a subgroup of G iff $a \in G$.
- iii) Define factor group and find all elements of the factor group $\frac{\mathbb{Z}}{5\mathbb{Z}}$.
- iv) Write $U(105)$ as an external direct product of cyclic groups.
- v) State the first isomorphism Theorem.
- vi) Define Maximal Ideal and find all the maximal ideals of $\mathbb{Z}(+, \times)$.
- vii) Let $f(x) = x^2 + 2x + 3$ & $g(x) = 2x^3 - 4x + 1$ then find the product $f(x)g(x)$ in the polynomial ring $\mathbb{Z}_5[x]$.
- viii) Test the polynomial $p(x) = 21x^3 - 3x^2 + 2x + 9$ is reducible or irreducible over $\mathbb{Q}$.
- ix) Define irreducible element in an integral domain.

3. Answer **EIGHT** of the following.

[8 x 5 = 40]

- i) Show that the Alternating Group A_n has order $\frac{n!}{2}$ for $n > 1$.
- ii) Prove that every group of prime order is cyclic.
- iii) If the pair of cycles $\alpha = (a_1, a_2, \dots, a_m)$ and $\beta = (b_1, b_2, \dots, b_n)$ have no entries in common, then show that $\alpha\beta = \beta\alpha$.
- iv) Let ϕ be a group homomorphism from a group G to a group $\bar{G}$. Then define kernel of a Homomorphism ($\text{Ker}\phi$) and show that $\text{Ker}\phi$ is a normal subgroup of G .
- v) Determine the number of elements of order 5 in $\mathbb{Z}_{25} \oplus \mathbb{Z}_5$.
- vi) List all the elements of the factor ring $\frac{\mathbb{Z}[i]}{\langle 2-i \rangle}$.

- vii) Let ϕ be a ring homomorphism from a ring R to a ring S . Then show that ϕ is an isomorphism if and only if ϕ is onto and $\text{Ker}\phi = \{r \in R \mid \phi(r) = 0\} = \{0\}$.
- viii) Find the quotient and remainder upon dividing $f(x) = 3x^4 + x^3 + 2x^2 + 1$ by $g(x) = x^2 + 4x + 2$, where $f(x)$ and $g(x)$ in $\mathbb{Z}_5[x]$.
- ix) Prove that for any prime p , the polynomial $f(x) = x^{p-1} + x^{p-2} + \dots + x + 1$ is irreducible over $\mathbb{Q}$.
- x) Show that the ring of Gaussian integers $\mathbb{Z}[i]$ is a Euclidean domain with $d(a + bi) = a^2 + b^2$.

4. Answer **FOUR** of the followings.

[4 × 8 = 32]

- i) Show that the order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles. By using it find the order of the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 1 & 3 & 6 & 2 & 4 \end{pmatrix}$.
- ii) State and prove Lagrange's Theorem.
- iii) Let G be a group and let $Z(G)$ be the center of G . Then show that If $G/Z(G)$ is cyclic, then G is Abelian. Use it to show that if $|G| = pq$, $p < q$ and G is non abelian then $|Z(G)| = 1$, where p, q are prime numbers.
- iv) Let R be a commutative ring with unity and let A be an ideal of R . Then show that R/A is an integral domain if and only if A is prime.
- v) State and prove Eisenstein's Criterion.

APRIL, 2026
CALCULUS AND ANALYTIC GEOMETRY

Full Marks: 100

Time: 3 hours

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1. Answer all the questions

(10x1=10)

- The derivative of $\sinh x$ is _____.
- The point where the concavity of a curve changes is called _____.
- $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x} + \sqrt{\sin x}} dx = \underline{\hspace{1cm}}$.
- If $m \neq n$, then $\int_0^{\pi} \cos mx \cdot \cos nx dx$ is _____.
- The type of the conic $x^2 - 4y^2 + 5x - 8y + 16 = 0$ is _____.
- Write the parametric equation of $9x^2 + 4y^2 = 36$.
- The unit vector in the direction of $3\hat{i} + 4\hat{j}$ is _____.
- The value of $[\hat{i} \hat{j} \hat{k}] = \underline{\hspace{1cm}}$.
- Three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ are collinear if _____ = 0.
- For vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$ the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$ gives the volume of the _____ formed by the three vectors.

2. Answer All the questions:

(9 × 2 = 18)

- State Leibnitz rule for the nth derivative of the product of two functions.
- Write the reduction formula for $\int \sec^n x dx$.
- Define concavity and inflection point of a curve.
- What is the condition for a point (a, b) to be a point of inflection for the curve $y = f(x)$?
- State L'Hospital's rule for evaluating $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ when it is of the form $\frac{0}{0}$.
- Define arc length of a curve given by $y = f(x)$ from $x = a$ to $x = b$.
- If the vector function $\vec{r}(t)$ have constant magnitude then show that $\vec{r} \cdot \frac{d\vec{r}}{dt} = 0$.
- Determine the constant a so that the vector $V = (x + 3y)\hat{i} + (y - 2z)\hat{j} + (x + az)\hat{k}$ is solenoidal.
- Define the divergence and curl of a vector function F .

3. Answer any Eight:

(8 × 5 = 40)

- Trace the curve $y^2(a + x) = x^2(a - x)$.
- Find the asymptotes of the curve $x^3 + y^3 = 3axy$.
- Evaluate $\int_0^{\pi} \frac{dx}{5 + 4 \cos x}$.
- Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x dx$ using the reduction formula.

- e. Find the volume of the solid generated by revolving the region bounded by $y=x^2$ and $y=4$ about the x-axis using the washer method.
- f. Find the arc length of the parametric curve $x = a(\theta - \sin\theta), y = a(1 - \cos\theta)$ for $0 \leq \theta \leq 2\pi$.
- g. Find the area of the surface generated by revolving the curve $y = \sqrt{x}$ from $x = 0$ to $x = 4$ about the x-axis.
- h. If $F(x, y) = u(x, y)\mathbf{i} + v(x, y)\mathbf{j}$, show that $\text{Curl } F = 0$ if and only if $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$.
- i. Find the constant a, b, c so that the vector $V = (x + 2y + az)\mathbf{i} + (bx - 3y - z)\mathbf{j} + (4x + cy + 2z)\mathbf{k}$ is irrotational.
- j. If $F = x^2y\mathbf{i} + y^2z\mathbf{j} + z^2x\mathbf{k}$, find $\text{div } F$ and $\text{curl } F$.

4. Answer any Four questions.

(4x8=32)

- a. Prove that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = \frac{\pi}{2} \log \left(\frac{1}{2} \right)$.
- b. Find the asymptotes of the curve:

$$x^3 - x^2y - xy^2 + y^3 - x^2 + y^2 - 10x - 2y + 1 = 0.$$
- c. Find the ranges of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also determine the point of inflections.
- d. Find the volume of the solid obtained by revolving the circle $x^2 + y^2 = 1$.
- e. The position of an object at time t is given by $(4 + \cos 3t, \sin 3t)$. Find the velocity, acceleration, and the normal and tangential components of acceleration of the object at time t .
