April,2026

Real Analysis-II

Time: 3 hours **Full marks:** 100

Answer as per instructions. All symbols have their usual meaning
Figures in the right-hand margin indicates marks.

1. Answer **ALL** the followings. **[10x1= 10]**

   i) The function $f(x) = x|x|$ is not differentiable at $x = 0$. (True/False)
   ii) For $f(x) = \sin x$ on $[0, \pi]$, find 'c' satisfying Rolle's Theorem.
   iii) If $f'(x) = 0$ for all x in an interval, what can you conclude about $f(x)$?
   iv) Every monotonic function on a closed interval is Riemann Integrable .(True /False)
   v) Give an example of a function f which is not Riemann Integrable but $|f|$ is Riemann Intergrable.
   vi) Find $\lim_{n\to\infty} \frac{1}{x+n}$ for all $x \in R$.
   vii) The radius of convergence of the power series $\sum_{n=1}^{\infty} x^{2n}$ is _________.
   viii) Is the integral $\int_0^1 \frac{dx}{\sqrt{x}}$ improper?
   ix) $\Gamma(6) =$__________. (Γ *is gamma funciton*)
   x) The integral $\int_1^2 \frac{1}{x\log x} dx$ is convergent. (True/False)

2. Answer **ALL** the followings. **[9x2 = 18]**

   i) State Darboux's Theorem.
   ii) Write the Taylor series of $\cos x$ about $x = 0$.
   iii) Verify Lagrange's Mean Value theorem for $f(x) = \log x$ on $[1, e]$
   iv) Define Riemann Sum.
   v) Evaluate $\int_3^4 \frac{dx}{x^3}$.
   vi) Write Weierstrass M-test for uniform convergence of sequence.
   vii) Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{x^n}{2^n}$.
   viii) Test for convergence of $\int_0^{\infty} \frac{1}{1+x^2} dx$.
   ix) Show that $B(m, n) = B(n, m)$ where B *is* beta function and $m, n > 0$.

3. Answer **EIGHT** of the following. **[8x5= 40]**

   i) State Lagrange's Mean Value Theorem and use the theorem to prove that
   $$|\sin x - \sin y| \leq |x - y| \text{ for all } x, y \text{ in } R.$$
   ii) Show that $\frac{x}{1+x^2} < \tan^{-1} x < x \; for\; all\; x > 0$.
   iii) Show that the function f defined by $f(x) = |x - 2| + |x - 1|$ is continuous but not differentiable at $x = 1 \; and \; 2$.

iv) Using Riemann sum show that $\int_a^b x^2 dx = \frac{b^3-a^3}{3}$.

v) Evaluate $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{k^2+n^2}$.

vi) Show that the sequence f_n where $f_n(x) = x^n$ is uniformly convergent on $[0, k], k < 1$ and only point wise on [0,1].

vii) Test the uniform convergence of the series $\sum \frac{sin(x^2+n^2x)}{n(n+1)}$ for all real x.

viii) Prove that the derivative of a power series has the exact same radius of convergence as the original series. Demonstrate it by $\sum_{n=1}^{\infty} x^n$.

ix) Examine the convergence of the improper integral $\int_0^2 \frac{dx}{x^2-4x+3}$.

x) Define absolute convergence and show that $\int_1^{\infty} \frac{\sin(x)}{x^3} dx$ convergence absolutely.

4. Answer **FOUR** of the followings **[4x8 = 32]**

i) State and prove Carathéodory's Theorem. Illustrate the theorem by considering the function $f(x) = x^3$

ii) State Taylor's Theorem and use the theorem to approximate the number e with error less than 10^{-5}. $(e = \text{Euler's number})$

iii) State and prove Fundamental theorem of Calculus.

iv) Show that the sequence $f_n(x) = nxe^{-nx^2}, x \geq 0$_ is not uniformly convergent on $[0, k], k > 0$.

v) Define Beta function ($B(m, n)$) and prove the convergence criteria for Beta function.

****

+3 4th SEM (NEP)
MAT-C-1P-9 (MAJ)
Arts/Sc.

April, 2026
Complex Analysis

Full Marks: 100 **Time: 3 hours**

Answer as per instructions. All symbols have their usual meaning
Figures in the right-hand margin indicates marks.

1. Answer the following. **(10 × 1 = 10)**

a) Find the modulus of the complex number $(-3-4i)$.
b) Write the polar form of complex number $(-1+i\sqrt{3})$.
c) A function that is analytic everywhere in the complex plane is called ______.
d) State Cauchy–Riemann equations in polar form.
e) sin z= 0 <=> z= _________.
f) What is a singular point?
g) A pole of order one is called a ______ pole.
h) What is a smooth curve?
i) An open connected set in the complex plane is called a ____.
j) Find all the values of $log(1-i)$.

2. Answer All. **(9 × 2 = 18)**

a) Show that a complex number z is purely imaginary iff $z + \overline{z} = 0$.
b) Calculate: $\int_C \frac{dz}{z}$, where C is the unit circle.
c) Calculate the value of i^i.
d) Locate and classify the nature of the singularity of $\frac{2(1-\cos\ )}{z^2}$.
e) State Morera's theorem.
f) State Laurent's theorem.
g) Calculate the residues of the function $f(z) = \frac{1}{(z-1)^2(z-3)}$.
h) When a function is said to have an isolated singularity at infinity? What is the nature of the singularity of $f(z) = z^2 + 1$ at $z = \infty$.
i) Determine the location and order of the zeros of $(z^4 - 16)^4$.

3. Answer **any Eight**. **(8 × 5 = 40)**

a) Express $z = \frac{(-1)+i\sqrt{3}}{1+i}$ in polar form and find $|z|$ and $\arg z$.
b) For any two complex numbers z_1, z_2 , show that $|z_1 + z_2|^2 = 2(|z_1|^2 + |z_2|^2)$.
c) If the argument of an analytic function is constant in some domain, then show that the function is constant in that domain.
d) Is the function $f(z) = sin\ z$ bounded in the complex plane? Justify your answer.
e) Find the centre and radius of convergence of the series

(i) $\sum_{n=1}^{\infty}(1+\frac{1}{n})^{n^2} z^n$ and (ii) $\sum_{n=0}^{\infty}\frac{n^n}{n!}(z+2i)^n$

f) Evaluate the integral $\int_C (x^2 - iy^2)dz$,where C is the parabola $y = 2x^2$ from the point (1,2) to (2,8)

g) Show that $\int_C \frac{dz}{(z^2+4)^2} = \frac{\pi}{16}$, $C: |z - i| = 2$.

h) State and prove Liouville's theorem.

i) Find the Taylor's series expansion of the function $\log(1 + z)$ about $z = 0$ and determine the region of convergence.

j) Find the analytic function $f(z) = U(x, y) + iV(x, y)$ whose real part is given by $U(x, y) = e^{-x}[(x^2 - y^2) \cos y + 2x \sin y]$.

4. Answer any **Four** **(4 × 8 = 32)**

a) For a complex number $a + ib$, find the corresponding point (x, y, z) on the Riemann Sphere. Hence find the corresponding point on the sphere for the complex number $z = 1 - i$.

b) If $V(x, y)$ is a harmonic conjugate of $U(x, y)$ then show that $U(x, y)$ needn't be the harmonic conjugate of $V(x, y)$.

c) Let $f(z)$ be analytic in a simply connected domain containing the simple closed contour C . If z_0 is inside$C$, then show that $f^{/}(z_0) = \frac{1}{2\pi} \int_C \frac{f(z)}{(z-z_0)^2} dz$.

d) (i) Find the Laurent's series for $\frac{z}{(z+1)(z+2)}$ about $1 < |z| < 2$.

(ii) Using residue theorem evaluate $\int_C z^2 e^{\frac{1}{z}} dz$, where C: $|z| = 1$.

e) Characterize all rational functions which have

(i) a pole of order k at ∞.

(ii) a removable singularity at ∞.

****

$+3$ 4th Sem (NEP)
MAT-C-1P-10 (MAJ)
Arts/Sci.

April,2026
Algebra 2

Full Marks: 100 **Time: 3 hours**

Answer as per instructions. All symbols have their usual meaning
Figures in the right-hand margin indicates marks.

NO.1 Answer all the Question. **10x1=10**

1. Find the number of elements of order 6 in $\mathbb{Z}_5 \; X \; \mathbb{Z}_9$
2. Define Kernel of a Group Action.
3. If G is a group of order 45 then find the number of 5-Sylow Subgroup.
4. Give an example of Non Simple Group.
5. Find the number of abelian Group(up to Isomorphic) are there of order 21.
6. Find the class equation of $G = (\mathbb{Z}_6, +_6)$
7. Write the conjugacy class of $x \; if \; x \in Z(G)$.
8. A Group of Order $p^2$is always abelian(True/False)
9. Subgroup of a Solvable Group is necessarily solvable(True/False)
10. Define Simple Module.

No. 2 Answer **any nine** Question. **9x2 =18**

1. State Fundamental Theorem of Finite Abelian Group.
2. Define Maximal Subgroup, Give an Example of It.
3. Define Composite Series of a Group.
4. Define Solvable Group.
5. Define Module, Give an example.
6. Define a Group Action of a group G on a set X.
7. When a Group Action is called Faithful.
8. Define Orbit of an element in a Group Action.
9. Define Simple Group; Give an example of Simple Group.
10. Find the first-Class equation of the Group S_3

No.3 Answer **Any Eight** Question. **8x5 = 40**

1. Use Fundamental Theorem to find the all the abelian group or order 1125.

2. Show that every abelian group is Solvable.
3. Show the intersection of Two Sub module over a ring R is again a Sub module.
4. Find conjugacy class of each element of D_4(The dihedral group of order 8)
5. Show that a group of order 20 can't be a simple group.
6. Show that in a group G, A p-sylow subgroup is unique if and only if it is a normal subgroup of G.
7. If H is a p- Sylow Subgroup of G and $x \in G$ then show that $x^{-1}Hx$ is also a p-sylow Subgroup of G.
8. For a Group G Show that $cl(g) = \{g\}$ if and only if $g \in Z(G)$
9. What do you mean by Stabiliser of an element? Show that stabiliser of an element $g \in G$ is a Subgroup of G.

10 Let H be a Subgroup of Group G and L_H be the set of all left coset of H in G, Define the Action of G on L_H by $g * xH = gxH$, Show that L_H is a G-set.

No. 4 Answer **any Four** Question. **4 x 8 =32**

1. State and Prove Sylow's First Theorem.
2. Show that $A_5(alternating\ Group)$ is a simple Group.
3. If G be a Finite Group acting on a Set X, then for any $x \in X$ show that

   $|Gx| = |G: G_x|$(Orbit Stabilizer Theorem)

   Where Gx is the orbit of x and G_x is the Stabilizer of$x$.
4. Show that a Group of order 99 is abelian
5. Let the Group G act on a set X, for each $g \in G$, the map $\sigma_g: X \to X$ given by $\sigma_g(a) = g.a$ then show that

   i) σ_g is a permutation on the set X.

   ii) The Mapping $\phi: G \to S_X$ given by $\phi(g) = \sigma_g$ is a permutation.

****

+3 4th Sem (NEP)
MAT-C-3P-2 (MIN)
Arts/Sci.

APRIL, 2026
Introduction to Algebra and Number Theory

Full Marks: 100 Time: 3 hours

Answer as per instructions. All symbols have their usual meaning
Figures in the right-hand margin indicates marks.

NO.1 Answer all the Question. **10x1 =10**

1. Write the unit element of $\mathbb{Z}_6$.
2. The set $(\mathbb{Z}, -)$ is a abelian Group(True/ False)
3. Give two example of non abelian group.
4. Give two example of Field
5. Define Dihedral group with example.
6. Define Fibonacci Number.
7. The integer 701 a Prime Number(True/False)
8. Find the reminder 15! is divided by 17.
9. $\mathbb{Z}_{10}$ is a field(True/False)
10. If $a = 2.3^2.5^4$ and $b = 2^5.3^2.7^2$ then find the $\gcd(a, b)$.

No. 2 Answer **All** the Question. **9x2 =18**

1. State fundamental theorem of algebra
2. Define Lucas Number, Give first Four Lucas Number.
3. If $a \mid b$ and $a \mid c$ then show that $a \mid bx + cy$ for all $x, y \in \mathbb{Z}$
4. If $g = \gcd(a, b)$ and $'d'$ is any common divisor of a and b then show that $'d'$ divides g
5. Is $\overline{17} \equiv \overline{5}(mod\ 6)$, Justify your answer.
6. Find the reminder when 16^{49} is divided by 7
7. Define normal subgroup , Give example
8. What is Zero divisor , Find all the zero divisor of $\mathbb{Z}_{10}$
9. Let $(R, +, X)$ is an integral Domain, for $a, b, c \in R$ and $a \neq 0$ if $ab = ac$ $then$ $show$ $that$ $b = c$

No.3 Answer **Any Eight** Question. **8x5= 40**

1. Show that there are infinitely many prime numbers.
2. If $a, b \in \mathbb{Z}$ and $\gcd(d, a) = 1$ and $d \mid ab$ then show that $d \mid b$.
3. Find all the solution of the equation $18x \equiv 30\ (mod\ 42)$
4. Prove that in a group$(G, *)$, the inverse of an element is unique.
5. Prove that Every Cyclic Group is Abelian group.
6. Show that every Finite integral domain is a field.
7. Show that the intersection of two sub ring is again a sub ring.
8. Solve the Liner Diophantine Equation $10x - 7y = 17$.
9. If $'a'$ and $'b'$ are two positive integers then show that $\gcd(a, b)\ lcm(a, b) = ab$
10. Find the Lcm (306,657)

No. 4 Answer **any Four** Question. 4 X 8 =32

1. State and prove Fermat little's Theorem.
2. Using Euclidean Algorithm, obtain $m, n \in \mathbb{Z}$ such that $\gcd(1769,2378) = 1769m + 2378\,n$
3. Using Chinese Reminder theorem solve the system $\begin{matrix} x \equiv 2\ (mod\ 5) \\ x \equiv 3\ (mod\ 7) \end{matrix}$
4. Let $G = GL(2, \mathbb{R})$ (Group of all 2 x 2 invertible Real Matrix) and $H = \left\{ \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \mid a, b \in \mathbb{Z} - \{0\} \right\}$, Prove or Disprove that H is a Subgroup G.
5. Describe the Following (Any Four)
   a) Integral Domain
   b) Cyclic Group with Example
   c) Normal Subgroup
   d) Field
   e) Centre of a Group.

****