+3 SEM-1 (NEP)
MAT-C-1P-1(R/B) MAJ
Arts/Sc

DECEMBER, 2025
CALCULUS AND ANALYTIC GEOMETRY

Full Marks: 100 **Time: 3 hours**

Answer as per instructions. All symbols carry their usual meanings
Figures in the right hand margin indicates marks

1. Answer all the questions **(10x1=10)**
   (i) What is the number of points in which any asymptote of a curve of nth degree cuts the curve?
   (ii) If $y = e^{mx}$ then what is the value of y_n?
   (iii) Which of the following is not an indeterminate form?
   (a) $\frac{\infty}{\infty}$ (b) $0.\infty$
   (c) ∞^0 (d) 0^∞
   (iv) $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\cos x}+\sqrt{\sin x}} dx = -$
   (v) $\int e^x[f(x)+f'(x)]dx$ is equal to
   (a) $\int e^x[f'(x)]dx$ (b) $e^x f(x) + C$
   (c) $e^x f'(x) + C$ (d) None of these
   (vi) If $m \neq n$,then $\int_0^\pi \cos mx . \cos nx \, dx$ is ___
   (vii) The type of conic $x^2 - 4y^2 + 5x - 8y + 16 = 0$ is__
   (viii) Write the parametric equation of $x^2 + 4y^2 = 36$
   (ix) The unit vector in the direction of $2\hat{\imath} + 2\hat{\jmath}$ is___
   (x) The value of $[\hat{\imath} \ \hat{\jmath} \ \hat{k}] =$ __

2. Answer all the questions. **(9x2=18)**
   (i) Express e^x in terms of $sinhx$ and $coshx$.
   (ii) Find the value of $\lim_{x\to 0} x^x$
   (iii) Prove that $\int_0^a f(x)dx = \int_0^a f(x-a)dx$
   (iv) Integrate: $\int_{-1}^{2} \frac{|x-2|}{2} dx$
   (v) Integrate: $\int e^{x^3} x^2 dx$
   (vi) Define a parabola. What is the eccentricity of the parabola?
   (vii) Find the arc length of the curve $x = t^2, y = t - \frac{t^3}{3}, -\sqrt{3 \leq t \leq \sqrt{3}}$

(viii) If the vector function $\overrightarrow{f(t)}$ have constant magnitude then show that $\vec{f}.\frac{\overrightarrow{df}}{dt} = 0$.

(ix) If $r = \vec{a}.e^{nt} + \vec{b}.e^{-nt}$ $where\ \vec{a}\ and\ \vec{b}$ are constant vectors show that

$\frac{d^2r}{dt^2} - n^2r = 0.$

3. Answer any **Eight** questions **(8x5=40)**

(i) If $y = \sin(m\sin^{-1}x)$ then show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0.$

(ii) Trace the curve: $r = a(1 - cos\theta)$

(iii) Evaluate $\int_0^{\frac{\pi}{2}} sin^5 x\, dx$ using a reduction formula.

(iv) Show that $\int \frac{x+\sin x}{1+\cos x} dx = x\, tan\frac{x}{2}$

(v) Find $\int_{-1}^{1}\{|x| + [x]\}\, dx$

(vi) Identify the conic $y = x^2 - 2x + 3$. Find its focus and vertex.

(vii) Using washer method, find the volume of the solid that results when the region enclosed by the curve $y = \frac{1}{x}, x = 2$ is resolved about Y-axis.

(viii) Calculate the arc length of the curve: $x = \cos^3 t, y = \sin^3 t, z = 2, 0 \le t \le \frac{3\pi}{4}$

(ix) Prove that $[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}\,]^2$.

(x) If $F(x,y) = u(x,y)i + V(x,y)j$, show that $Curl\ F = 0$ if and only if $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

4. Answer any **Four** questions. **(4x8=32)**

(i) Prove that $\int_0^{\frac{\pi}{2}} \log sinx\, dx = \frac{\pi}{2}\log\left(\frac{1}{2}\right)$.

(ii) Find the asymptotes of the curve:
$x^3 - x^2y - xy^2 + y^3 - x^2 + y^2 - 10x - 2y + 1 = 0.$

(iii) Find the ranges of values of x for which the curve $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also determine the point of inflections.

(iv) Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

(v) The position of an object at time t is given by$(4 + \cos 3t, \sin 3t)$. Find the velocity, acceleration, the normal and tangential components of acceleration of the object at time t.

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+3 SEM-1 (NEP)
MAT-C-1P-2(R/B) MAJ
Arts/Sc

DECEMBER, 2025

Introduction to Algebra and Number Theory

Full Marks: 100 **Time: 3 hours**

NO.1 Answer all the Question. **10x1 =10**

1. Define Dihedral Group.
2. Define Centre of a group G, find the centre of D_4.
3. Find the order of the element 6 in $\mathbb{Z}_{20}$
4. Find all the generator of the cyclic Group $(\mathbb{Z}_{20}, +_{20})$
5. Write a normal subgroup of D_4.
6. $\mathbb{Z}_{10}$ is an integral domain. (True/False)
7. Write first four LUCAS Number.
8. Find the $GCD(a, b)\ if\ b|a$.
9. Find the reminder when 3^{161} is divided by 17 .
10. Determine whether the Linear Diophantine Equation $6x + 12y + 15z = 10$ is solvable or not?

No. 2 Answer All the Question. **9x2 =18**

1. Find the order of each element of group $(\mathbb{Z}_5, +_5)$?
2. If every element of a group G has self-inverse, then show that G is abelian.
3. Show that every Cyclic Group is Abelian.
4. Define integral domain, Give an example.
5. Define the Ring $\mathbb{Z}[i]$.
6. Show that for $K \in \mathbb{N}$, $\gcd(3K + 2, 5K + 3) = 1$.
7. Write Four consecutive integer that are composite number.
8. The LCM of Two Consecutive integer is 812, find them
9. Find all the prime number less than 25.

No.3 Answer **Any Eight** Question. **8x5 = 40**

1. If H and K are subgroups of Group G then Prove that $H \cup K$ is a subgroup of G if and only if $H \subseteq K\ or\ K \subseteq H$.
2. Let $G = (\mathbb{C}, +)$ be a group, let $H = \{a + ib : a, b \in \mathbb{R}, ab \geq 0\}$, Prove or disprove that H is a subgroup of G.
3. Show that $o(a) = o(a^{-1})$.
4. Let R be a ring with $a^2 = a\ \forall\ a \in R$ then show that R is a commutative Ring.
5. Express the $\gcd(2076, 1776)$ as a linear combination of the number $2076\ and\ 1776$.
6. Solve the congruence equation $12x \equiv 48\ (mod\ 18)$
7. Solve the linear Diophantine Equation $12x + 16y = 20$. Find the general solution and also find all the positive solution.
8. Using Chinese Reminder theorem solve the system $x \equiv 1(mod\ 3)$, $x \equiv 2(mod\ 5)$, $x \equiv 3\ (mod\ 7)$
9. Show that there are infinitely many primes.
10. If $a, b \in \mathbb{Z}\ and\ a = bq + r$ then show that $\gcd(a, b) = \gcd\ (b, r)$

No. 4 Answer **any Four** Question. **4 X 8 =32**

1. Show that a subgroup of a cyclic group is cyclic
2. Let $H = \{A \in GL_2(\mathbb{R}) |\det A\ \ is\ an\ inegral\ power\ of\ 2\}$, Show that H is a Subgroup of G, Check H for Normal subgroup of G.
3. State and proof Fundamental Theorem of Algebra.
4. State and Proof Wilson's Theorem.
5. If $g = \gcd(a, b)$ then prove that there exist $m, n \in \mathbb{Z}\ such\ that\ g = am + bn$

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+3 SEM-1(NEP)
MAT-MDC-1(R&B)

DECEMBER,2025
DISCRETE MATHEMATICS

Time: 3 Hours **Maximum Marks: 100**

Answer as per instruction.
figures in the righthand margin indicates marks

Section A

1. Answer all **(10x1 = 10)**

(i) Define a proposition.

(ii) Write the negation of the statement: "All integers are even."

(iii) State the Pigeon-hole principle.

(iv) How many subsets does a set with 3 elements have?

(v) Define a recurrence relation.

(vi)What is a partially ordered set?

(vii) Write the formula for $C(n, r)$.

(viii) Define a loop in a graph.

(ix) What is a simple graph?

(x) State Euler's condition for existence of an Euler circuit.

Section B

2.Answer all **(9 × 2= 18)**

(i) Write the truth table of $p \rightarrow q$.

(ii) Define reflexive and symmetric relations with examples.

(iii) How many ways can 3 objects can be chosen from 7 objects?

(iv) Solve: $a_n = a_{n-1} + 2$, $a_1 = 1$. Find a_3.

(v) Define a chain in a poset.

(vi) Draw the Hasse diagram of {1, 2, 4} under divisibility.

(vii) What is a connected graph? Give an example of a connected graph.

(viii) Define a path and a circuit in a graph.

(ix) State the handshaking lemma.

Section C

3. Answer **any Eight** **(8 × 5 = 40)**

(i) Prove $(p \wedge q) \rightarrow p$ is a tautology.

(ii) Show that the congruence modulo relation is an equivalence relation on the set of integers.

(iii) If $P(5, r) = 2 \times P(6, r-1), find\ r.$

(iv) How many arrangements can be made with the letter of the word 'MATHEMATICS'.

(v) Solve $a_n = 3a_{n-1}$, $a_0 = 2$.

(vi) Define lattice and explain meet and join.

(vii) A committee of 5 is to be formed out of 6 gents and 4 ladies. In how many ways this can be done so that at least 2 ladies are included.

(viii)Define degree of a vertex in a graph. How many edges are there in a graph with 10 vertices each of degree six?

(ix) Show that $\sim\big(p \vee (\sim p \wedge q)\big)$ and $(\sim p) \wedge (\sim q)$ are logically equivalent.

(x) Define the chromatic number of a graph. Determine the chromatic number of the complete bipartite graph $K_{m,n}$ where m and n are positive integers?

Section D

4. Answer **any Four** **(4 × 8 = 32)**

(i)Prove that √2 is irrational.

(ii) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5, a_2 = 15$

(iii) Draw Hasse diagram and explain duality.

(iv)Explain Euler and Hamiltonian paths and circuits with examples.

(v)Explain planar graphs. Is $K_{3,3}$ planar? Justify your answer.

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+3 SEM-1 (NEP)
MAT-C-2P-1(R/B) MIN
Arts/Sc

DECEMBER, 2025

Calculus and Analytic Geometry

Full Marks:100 **Time: 3 Hours**

1. Answer **ALL** the questions. **[$10x1 = 10$]**

i) Find the value of $\tan h(\ln 4)$.

ii) Write the vertical asymptotes of the curve $xy - 2y = 1$.

iii) The curve $x^3 + y^3 = 3axy$ is symmetrical about________.

iv) Integrate $\int xe^x dx$.

v) $\int_0^{\frac{\pi}{2}} \sin^3 x\, dx =$________.

vi) Classify the conic $4x^2 - 4xy + y^2 - 8x + 5y - 7 = 0$.

vii) For the ellipse $9x^2 + y^2 = 9$, the foci are__________.

viii) Write the parametric equation of parabola.

ix) Find $u.(v \times w)$ if $u =< 1, -2, 2 >$, $v =< 0,3,2 >$, $w =< -2,4,-4 >$.

x) $\lim_{t\to 0} \frac{\hat{\imath}+t\hat{\jmath}-e^{-t}\hat{k}}{1-t} =$________.

2. Answer **ALL** the questions. **[$9x2 = 18$]**

i) Find $\frac{d^2y}{dx^2}$, if $x = t^2, y = 2t$.

ii) Find the vertical and horizontal asymptote of $f(x) = \frac{1}{x^2-9}$.

iii) Evaluate $\lim_{x\to\frac{\pi}{2}} \sin x^{\tan x}$.

iv) Explain the meaning of $\int_a^b f(x)\, dx$ as a limit of a sum.

v) Evaluate $\int x^2 \cos x\, dx$

vi) Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 2$ and $x = 0$ revolved about y-axis.

vii) Write the formula for finding the arclength of the curve.

$$x = t - \sin t,\ y = 1 - \cos t,\ 0 \le t \le 2\pi$$

viii) If R is a position vector of a moving body, then find the velocity V and the acceleration A, where $R(t) =< t\, sint, e^{-t}, t >$.

ix) Evaluate $\int_0^1 \left[\frac{2}{\sqrt{t}}\hat{\imath} + \frac{\sqrt{3}}{1+t}\hat{k}\right] dt$.

3. Answer any **EIGHT** the questions. **[$8x5 = 40$]**

i) Find the asymptotes of the following curve:

$$2x^3 - x^2y + 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0$$

ii) Find the concavity and inflection point of the function f defined by

$$f(x) = \frac{x^3}{3} + x^2 - 15x + 3$$

iii) The total cost function for producing a certain product in x quantity is $C(x) = 60 - 12x + 2x^2$. Find the average cost function and level of production that minimizes this function.

iv) Evaluate $\int \sin^6 x \cos^5 x \, dx$.

v) Find the surface area generated when the graph of the function given by $f(x) = \frac{1}{3}(12 - x)$ on [0,3] is revolved about y-axis.

vi) Find the volume of the solid that results when the region in enclosed by the given curve $y = x^2, y = 2x$ revolved about x-axis.

vii) Explain the reflection property of Parabola.

viii) The base of a solid is the region bounded by $y = x$ and $y = 0$ for $0 \le x \le 2$. If cross-sections perpendicular to the x-axis are semicircles, find the volume.

ix) Find a vector function F such that $F''(t) = e^t\hat{\imath} - t^2\hat{\jmath} + 3\hat{k}$, $F(0) = \hat{\imath} - 2\hat{\jmath}$, $F'(0) = 3\hat{k}$.

x) Find the tangential and normal components of the acceleration of an object that moves with position vector $R(t) = < \frac{5}{13}\cos t, \frac{12}{13}(1 - \cos t), \sin t >$

4. Answer any **FOUR** the questions. **[$4x8 = 32$]**

i) Sketch the graph of $f(x) = \frac{3x-5}{x-2}$.

ii) If $y = xsin^{-1}x$, prove that $(1 - x^2)y_{n+2} - (2n + 3)xy_{n+1} - (n + 1)^2 y_n = 0$.

iii) Derive the reduction formula for $\int \cos^n x \, dx$. Use it to evaluate $\int_0^{\pi/2} \cos^4 x \, dx$.

iv) Sketch the graph of $9x^2 + 4y^2 - 18x + 24y + 9 = 0$. Also identify the center, major axis, minor axis, end points of major and minor axis, foci, vertex and x-intercept.

v) Find the value of a which satisfies the equation

$$\int_0^a \left[\left(t\sqrt{1 + t^2}\right)\hat{\imath} + \left(\frac{1}{1+t^2}\right)\hat{\jmath} \right] dt = \frac{1}{3}\left(2\sqrt{2} - 1\right)\hat{\imath} + \frac{\pi}{4}\hat{\jmath}$$

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