

April,2025

Electricity and magnetism (Major)

Full Marks-100

Time: 3 Hours

1. Answer the following questions **10×1=10**
- a) The SI unit of permittivity is _____
i) Nm^2C^{-2} ii) Nm^{-2}C^2 iii) $\text{N}^{-1}\text{m}^{-2}\text{C}^2$ iv) $\text{N}^{-1}\text{m}^2\text{C}^{-2}$
- b) An electron is brought towards another electron. The electric potential energy of the system:
i) Decreases ii) Increases iii) Remains the same iv) Becomes zero
- c) Over the equipotential surface, the electric field intensity is:
i) Everywhere normal to the surface
ii) Everywhere tangential to the surface
iii) Depending on the nature of the charge distribution
iv) None of the above
- d) $\vec{\nabla} \cdot \vec{D} = \rho$ based on
i) Faraday's Law ii) Ampere's Law iii) Ohm's Law iv) Gauss Law
- e) The relation $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$ holds good:
i) Only vacuum ii) Only inside a dielectric iii) only outside the dielectric
iv) Everywhere
- f) Two current-carrying conductors are parallel to each other. The currents are flowing in the same direction inside them. They will experience
i) No mutual force ii) Repulsion iii) Attraction iv) None of these
- g) In a ballistic galvanometer:
i) The moment of inertia of the coil must be large ii) The torsional constant of the suspension must be small
iii) Coil should be wound on a non-conducting frame iv) All of the above
- h) Kirchoff's current law is based on the principle of conservation of:
i) Charge ii) Energy iii) Momentum iv) Mass
- i) Series resonance occurs when inductive reactance is equal to:
i) Zero ii) Capacitive reactance iii) Reciprocal of capacitive reactance
iv) Square root of capacitive reactance
- j) Quality factor of an LCR circuit is given by:
i) ωL
ii) $\frac{L}{R}$
iii) $\frac{\omega L}{R}$
iv) $\frac{\omega}{R}$

2. Answer the following questions

9×2=18

- a) What is electric flux?
- b) State the coulomb law and express it in vector form?
- c) Write the boundary conditions satisfied by E and D at the boundary of two dielectrics.
- d) What do you understand by electronic, ionic, and dipolar polarization?
- e) The electric field in a certain region of space is given by $5\hat{i} + 4\hat{j} + 3\hat{k}$. Calculate the electric flux through a surface of area 100 units lying in the x-y plane
- f) Write the Bio-Savart Law in vector form, explaining the symbols.
- g) Define logarithmic decrement.
- h) Define self-inductance and mutual inductance?
- i) State the compensation theorem?

3. Answer any eight from the following questions:

8×5=40

- a) Use Gauss's theorem to show that the electric strength outside a charged sphere is the same as if the charge were concentrated at the centre.
- b) Describe the electric displacement vector? Deduce the Gauss law for a dielectric?
- c) What do you mean by magnetic vector potential? Obtain an expression for the same due to a current loop?
- d) Find the mutual inductance between two coplanar concentric rings of radii R_1 and R_2 , respectively?
- e) State and prove Norton's theorem in a network?
- f) Why series circuit is called an acceptor circuit? Plot variation of impedance, reactance and current with frequency for a series circuit?
- g) State and prove the reciprocity theorem of mutual inductances?
- h) What do you mean by electromagnetic damping in a ballistic galvanometer? Distinguish between charge sensitivity and current sensitivity of a ballistic galvanometer?
- i) Find the magnetic vector potential for two long straight parallel wires carrying current in opposite directions?
- j) Show that $E = -\vec{\nabla}V$ where E is the electric field and V is the electric potential?

4. Answer any **four** from the following question

4×8=32

- a) Find the expression for the electrostatic potential energy of a charged sphere.
- b) The capacitance of a parallel plate capacitor is 400 picofarad, and its plates are separated by 2 mm of air. (i) What will be the energy when it is charged to 1500 volts? (ii) What will be the potential difference with the same charge if the plate separation is doubled?
- c) Explain the construction of a ballistic galvanometer and find an expression for its charge sensitivity?
- d) Find the expression for the energy loss in a hysteresis cycle?
- e) State and explain the maximum power transfer theorem?

April,2025
Mathematical Physics-II(Major)

Full Mark - 100

Time -3 Hours

PART I: Answer all questions

[1x10]

1. The Fourier series of an even function contains only
 - a. Cosine terms
 - b. Sine terms
 - c. Both sine and cosine terms
 - d. Exponential terms
2. The complex exponential form of the Fourier series for a periodic function $f(x)$ with period 2π is
 - a. Exponential
 - b. Trigonometric
 - c. Orthogonal
 - d. Coefficient
3. A function $f(x)$ is said to be periodic if
 - a. $f(x+1)=f(x)$ for all x
 - b. $f(x+T)=f(x)$ for some $T>0$ and all x
 - c. $f(x)=-f(x)$
 - d. $f(x+2\pi)=f(x)$ only for trigonometric functions
4. The Legendre differential equation has regular singular points at
 - a. $x = 0$ and $x = 1$
 - b. $x = \infty$ only
 - c. $x = \pm 1$
 - d. $x = 0$ only
5. The indicial equation in the Frobenius method is obtained by:
 - a. Setting the coefficient of the highest derivative to zero
 - b. Substituting a power series solution into the ODE and equating the lowest power of x to zero
 - c. Integrating the ODE once
 - d. Ignoring singularities
6. Which formula expresses Legendre and Hermite polynomials using derivatives?
7. Which modified form of Legendre polynomials appears in spherical harmonics?
8. What is the relation between the Beta and Gamma functions?
9. When solving the heat equation on a rod of length L with homogeneous boundary conditions $u(0, t) = u(L, t) = 0$, the spatial part $X(x)$ satisfies:
 - a. $X'' + \lambda X = 0$ with $X(0) = X(L) = 0$, giving eigenvalues $\lambda_n = (n\pi/L)^2$
 - b. $X'' - \lambda X = 0$ with $X(0) = X'(L) = 0$
 - c. $X'' + \lambda X = 0$ with $X'(0) = X'(L) = 0$
 - d. $X'' = 0$ with $X(0) = X(L) = 0$
10. What method involves expressing the solution of a partial differential equation as a product of single-variable functions to simplify the equation into ordinary differential equations?

PART-II: Answer all questions

[2x9]

1. State and explain Dirichlet Conditions.

- Determine the period of $\sin x + \cos x$
- Use Rodrigues' formula to evaluate $H_4(x)$
- Establish the relation between $H_n(x)$ and $H_n(-x)$ using the generating function.
- Evaluate the integral $\int_0^1 e^{-x^2} dx$ in terms of the error function $\text{erf}(x)$.
- Prove that beta function is symmetric in m and n .
- Prove that $\int_{-\infty}^0 e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$.
- Verify that the function $P_1'(x) = -\sqrt{1-x^2}$ is a solution of the Associated Legendre Differential Equation for $l=1, m=1$.
- Solve $\frac{\partial u}{\partial x} = 6 \frac{\partial u}{\partial y} + u$ using the method of separation of variables if $u(x, 0) = 10e^{-x}$

PART-III: Answer any eight questions

[5x8]

- Determine the value of $\sum_{n=1}^{\infty} \frac{1}{n^2}$ using the Fourier series.
- Find the Fourier series to represent a function $f(x) = \frac{1}{4}(\pi - x)^2, 0 < x < 2\pi$.
- Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$
- Express $4x^3 + 6x^2 + 7x + 2$ in terms of Legendre polynomials.
- Show that $J_1''(x) = -J_1(x) + \frac{1}{x}J_2(x)$
- Prove that $P_{2n+1}(0) = 0$.
- Explain error function and the complementary error function.
- Find the value of $\int_0^1 \frac{35^3}{32\sqrt{1-x}} dx$.
- Use the method of separation of variables to solve $4 \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 3u$,
Given, $u = 3e^{-x} - e^{-5x}$
- What is the solution of Laplace's equation in two dimensions?

PART-IV: Answer any four questions

[8x4]

- Find the Fourier series to represent the function $f(x) = |\sin x|$, for $-\pi < x < \pi$.
- Solve in series $2x(1-x) \frac{d^2y}{dx^2} + (1-x) \frac{dy}{dx} + 3y = 0$.
- Show that $P_n(x)$ is the coefficient of z^n in the expansion of $(1-2xz+z^2)^{-1/2}$ in ascending power of z .
- Evaluate $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$.
- A tightly stretched string flexible string has its ends fixed at $x=0$ and $x=l$, the string is given a shape defined by $F(x)=\mu x(l-x)$, where μ is a constant, and then released. Find the displacement of any point x of the string at any time $t>0$.

April,2025
Mechanics(Minor)
Part-I
Objective questions

Answer All Questions

(10 x 1)

- 1 (i) Gravitational potential due to a point mass is
(a) $+ Gm/r^2$ (b) $- Gm/r^2$ (c) $+ Gm/r$ (d) $- Gm/r$
- (ii) The S.I. Unit of surface tension is
(a) Nm (b) Nm^{-2} (c) Nm^{-1} (d) $N^{-1}m^{-2}$
- (iii) Bernoulli's theorem is basis on
(a) Conservation of momentum (b) Conservation of energy
(c) Conservation of mass (d) Mass-energy equivalence
- (iv) The period of oscillation of a torsion pendulum is independent of _____.
(a) its mass (b) rigidity modulus (c) acceleration due to gravity (d) none of these
- (v) The uniform bending, the elevation (y) is related to Young's modulus (q) by the relation
(a) $y \propto q$ (b) $y \propto 1/q$ (c) $y \propto q^2$ (d) none of these
- (vi) A satellite revolves round a planet of radius R in time T . What will be the period of revolution around another planet of radius $3R$?
(a) $3T$ (b) \sqrt{T} (c) $3\sqrt{3}T$ (d) $9T$
- (vii) Lorentz transformation assume
(a) Space and time both are relative (b) Space and time both are absolute
(c) Space is absolute and time is relative (d) Space is relative and time is absolute
- (viii) Gravitational forces are
(a) strong (b) weak (c) short range (d) non-central
- (ix) If an object reaches the speed of light, its length changes to _____?
(a) Zero (b) Infinite (c) Double of its value (d) Half of its value
- (x) Michelson Morley experiment proved that
(a) Earth is an Inertial Frame (b) Earth is a non-Inertial Frame
(c) There is no absolute frame of reference called ether in universe (d) None of these

PART-II

VERY SHORT TYPE

2. ANSWER ALL QUESTIONS

(9 x 2)

- (i) Derive an expression for the bending moment of a bar.
- (ii) Write a short note on Stress-Strain diagram.
- (iii) Show that the depression of cantilever is proportional to the cube of its length.
- (iv) Explain the variation of viscosity of a liquid with temperature.
- (v) Define moment of inertia of a rotating body.
- (vi) What is the physical significance of logarithmic decrement of a damped oscillation?
- (vii) What are central and non central forces? Give some examples.

- (viii) State and explain Bernoulli's theorem.
(ix) Is the second law of motion $\mathbf{F} = m\mathbf{a}$ always valid in special theory of relativity? Explain.

PART-III
SHORT TYPE

3. ANSWER ANY 8 OUT OF 10 QUESTIONS **(8 x 5)**

- (i) Deduce Stokes's formula for the velocity of a small sphere falling through a viscous liquid using the method of dimensions.
(ii) Find the expression for Poisson's ratio in terms of elastic constants.
(ii) Explain the determination of rigidity modulus using torsion pendulum.
(iii) Explain what is sharpness of resonance.
(iv) What is cantilever? Derive an expression for the oscillations of a cantilever.
(v) Obtain an expression for the kinetic energy of a rotating body.
(vi) What is Gravitational constant and what are its dimensions?
(vii) State and prove Kepler's II law.
(viii) The motion of a particle under the influence of a central force is described by $r = a \sin \theta$. Find the expression for the force.
(ix) Show that $x^2 + y^2 + z^2 - c^2 t^2$ is invariant under Lorentz transformations.
(x) An atomic clock is placed in a jet airplane. The clock measures a time interval of 3600 s when the jet moves with speed 400 m/s. How much larger a time interval does an identical clock held by an observer at rest on the ground measure?

PART-IV

LONG TYPE ANSWER ANY 4 OUT OF 5 QUESTIONS **(4x8)**

- 4 (i) Derive the Lorentz transformation equations.
(ii) Discuss the relativistic Doppler Effect and derive the relations for longitudinal and transverse Doppler Effect.
(iii) Derive the relation between elastic constants with usual meanings.
(iv) Derive the Poiseuille's formula for the coefficient of viscosity of a liquid.
(v) State and prove Kepler's law of planetary motion. Show that the square of the time period of revolution of a planet is proportional to the cube of semi-major axis of the orbit.
