

December, 2 0 2 5
(Mathematical Physics–III)

Full Marks : 100

Time : 3 hours

The figures in the right-hand margin indicate marks

Answer **all** questions

Part—I

1. Answer the following questions in 1 word each : 10×1

(a) The complex conjugate of the complex number $z = \text{---}$.

(b) If w is the cube root of unity, then $1 + w + w^2 = \text{---}$.

(c) The functions satisfying Laplace's equation are known as --- .

(d) The value of $\int_C \frac{dz}{z+2}$ for $|z|=1$ is

- (i) 1 (ii) 0 (iii) -1 (iv) ∞

(e) The Fourier transform of Dirac-Delta function $F\{\{\delta x\}\}$ is

- (i) $\frac{1}{\pi}$ (ii) $\frac{1}{2}$ (iii) $\frac{1}{\sqrt{2\pi}}$ (iv) $\frac{1}{2\pi}$

(f) If $f(x)$ is the Fourier transform of $f(x)$, then the Fourier transform of $F(at)$ is

- (i) $aF(xa)$ (ii) $\frac{1}{a}F\left(\frac{x}{a}\right)$ (iii) $\frac{1}{a}F\left(\frac{1}{ax}\right)$ (iv) $xF\left(\frac{x}{a}\right)$

(g) The Fourier transform of the convolution of $f(t)$ and $g(t)$ is equal to

- (i) product of their Fourier transform
(ii) sum of their Fourier transform
(iii) quotient of their Fourier transform
(iv) difference of their Fourier transform

(h) The value of $L[e^{-at}] = \text{_____}$.

- (i) $\frac{1}{s-a}$ (ii) $\frac{1}{s+a}$ (iii) $\frac{s-a}{s+a}$ (iv) 1

(i) The Fourier transform of $e^{-x^2/2}$ simply repeats itself.
(State True or False)

(j) $L^{-1}\left[\frac{1}{s-2} - \frac{1}{s}\right] = \text{_____}$.

- (i) e^{3t+1} (ii) e^{-3t+1} (iii) $e^t - 3$ (iv) 0

Part—II

2. Answer *all* questions within 50 words each : 9×2

(a) Express $\frac{2+4i}{1-i}$ in the form of $x+iy$.

(b) Find the poles for the function $f(z) = \frac{z}{\cos z}$ and state the nature of the poles.

(c) Obtain the Taylor series expansion of $f(z) = \cos z$ at $z = 0$.

(d) Write Dirichlet's conditions for Fourier transform.

(e) Evaluate $\int_0^\pi x \cos nx \, dx$.

(f) Find Fourier sine transform of first derivative.

(g) State and prove the change of scale property of Fourier transform.

(h) Define Laplace transform.

(i) Determine $L[t^4\delta(t-2)]$.

Part—III

3. Answer *any eight* questions within 250 words each : 8×5

(a) Use De-Moivre's theorem to solve $x^7 = 1$.

(b) Check the function $f(z) = z^{-1}$ is analytic or not.

(c) Find the poles and evaluate the residues of $f(z) = \frac{z^2}{z^2 + 4}$.

(d) Locate and classify the singularities of the function

$$f(z) = (z-2) \sin\left(\frac{1}{z+2}\right)$$

(e) State and prove Cauchy integral formula.

(f) Derive Fourier cosine transform.

(g) State and prove convolution theorem.

(h) Using Fourier transform, solve the equation

$$\frac{\partial u}{\partial t} = k \frac{\partial^2 u}{\partial x^2}, 0 < x < \infty, t = 0$$

subject to conditions (i) $u(0, t) = 0, t > 0$, (ii) $u(x, 0) = e^{-x}, x > 0$ and (iii) u and $\frac{\partial u}{\partial x}$ both tend to zero as $x \rightarrow \pm \infty$.

(i) Find Laplace transform of $\sin 3t \cos 2t$.

(j) Find expression for n th derivative of Laplace transform.

Part—IV

4. Answer *any four* questions within 800 words each : 4×8

(a) State and prove Cauchy-Riemann conditions for a function to be analytic.

(b) State and prove Laurent's expansion of a complex function.

(c) State and prove Fourier integral theorem.

(d) Using Fourier transform to find the solution of differential equation of damped harmonic oscillator.

(e) Using Laplace transform, find expression for current in a series R - L circuit.

DECEMBER -2025
WAVES AND OPTICS

Time: 3 Hrs

Full Marks: 100

Answer as per instructions.
Figures in the right-hand margin indicates marks

Part-I

1. Answer All questions

[10x1=10]

- a) Describe Nodal points of thick lens.
- b) What type of lens is used to correct the myopia of human eye?
- c) Write different types of wavefront possible.
- d) What is ripple wave?
- e) What is Unpolarized, monochromatic light?
- f) If $E_x = E_0 \cos(\omega t - x)$, $E_y = E_0 \sin(\omega t - x)$ then the nature of the wave is _____ .
- g) The highest order of maxima which may be observed with monochromatic light of wavelength 5000 \AA by means of grating with 6000 lines/cm is _____ .
- h) If two source distance is double and source and screen distance is half than the fringe width is become _____ .
- i) Give an example experiment which based on application of division of amplitude.
- j) If the size of objective lens of a telescope is double, the resolving power of the telescope become _____ .

Part-II

2. Answer All questions

[9x2=18]

- a) Draw the ray diagram of Lloyd's mirror experiment with proper labeling.
- b) The lines per inch of a grating plate is double then calculate change in resolving power of the grating.
- c) Give a comparison between reflecting and transmitting Newton's Ring.
- d) What do you mean by division of wave front and division of amplitude?
- e) How the source distance is measure in Fresnel's Bi-prism experiment? Elaborate.
- f) Two thin convex lenses with powers 50 cm each are placed coaxially 10 cm apart. Find the combine focal length and position of principal points P_1 and P_2 .
- g) Discuss the intensity distribution of spectrum of plane Diffraction grating.
- h) What is destructive interference? When it occurs?
- i) How does a longitudinal wave transport energy, and what quantity is transferred?

Part-III

3. Answer Any Eight

[8x5=40]

- a) In Newton's ring experiment, the diameter of 4th dark ring is 0.3cm and the diameter of the 15th dark ring is 0.58cm. Find the radius of the Plano-convex lens if the wavelength of the light used is 600nm. Using this information calculate the difference between diameter of 9th and 10th dark ring.
- b) What is the condition for diffraction? Distinguish between Fresnel and Fraunhofer diffraction.

- c) What will happen if the Sodium source is replaced with Mercury source in Lloyd's mirror experiment?
- d) Distinguish between spatial and temporal coherence.
- e) What is the radius of the first half period zone in a zone plate behaving like a convex lens of focal length 50cm for light of wave length 600nm ? Also draw the ray diagram.
- f) State and explain Huygens Principle.
- g) State and elaborate Fermat's principle.
- h) Distinguish between Longitudinal and Transverse Waves.
- i) What is wave velocity how it is related to particle acceleration.
- j) Describe the construction of Fabry-Perot interferometer.

Part-IV

4. (a) Determine Cardinal points and cardinal planes for two thin lens separate by a distance d which is less than focal point of the two thin lens. [8]

Or

- (b) Write short note on [4+4]
- i. Ramsden eyepiece
 - ii. Huygens eyepiece

5. (a) Elaborate both Graphical and Analytical Methods for Superposition of two perpendicular Harmonic Oscillations. [8]

Or

- (b) Write short note on [4+4]
- i. Lissajous Figure
 - ii. Pressure of a Longitudinal wave

6. (a) Discuss construction and working principle of Michelson's Interferometer. [8]

Or

- (b) Derive the necessary equation for maxima and minima for interference due to reflected light in thin films. [8]

7. (a) Derive the necessary equation for maxima and minima due to Fresnel diffraction at straight edge. [8]

Or

- (b) Elaborate the Fraunhofer diffraction at Circular aperture and give that if diameter of the aperture increases the radius of the central ring decreases. [8]

DECEMBER -2025

THERMAL PHYSICS

Full Mark – 100

Time – 3 Hours

Answer as per instructions.
Figures in the right-hand margin indicates marks

Part I

1. Answer the following questions **(10x1)**
- (a) State zeroth law of thermodynamics.
 - (b) Write any one limitation of first law of thermodynamics.
 - (c) Define the principle of increase of entropy.
 - (d) What is internal energy?
 - (e) What are extensive and intensive thermodynamic variables?
 - (f) What is transport phenomenon in gases?
 - (g) What is the significance of Clausius-Clapeyron equation?
 - (h) State law of equipartition of energy.
 - (i) Define temperature of inversion.
 - (j) What is Joule-Thomson effect?

Part II

2. Answer the following question in 50 words **(9x2)**
- (a) State the principle of unattainability of absolute zero.
 - (b) State conditions for reversibility of a process.
 - (c) A gas is allowed to expand adiabatically from 10 litres to 30 litres at an initial temperature of 300K. Calculate the work done.
 - (d) Differentiate between 1st order and 2nd order phase transition.
 - (e) Prove, using Maxwell's thermodynamic relations, $\left(\frac{\partial C_V}{\partial V}\right)_T = 0$ for a real gas.
 - (f) Using the expression of coefficient of viscosity, discuss its dependence on pressure and temperature of the gas.
 - (g) Calculate the coefficient of performance of Carnot's refrigerator working between the temperatures 127°C and 27°C.
 - (h) The Vander Waal's constants of H₂ are $a = 2.47 \times 10^{-2} \text{ Nm}^4/\text{mol}^2$ and $b = 26.5 \times 10^{-6} \text{ m}^3/\text{mol}$. Calculate Boyle's temperature.
 - (i) Why do H₂ and He show heating effect at ordinary temperature?

Part III

3. Answer any 8 of the following in 250 words: (8x5)

- (a) Establish the Clausius inequality theorem.
- (b) Prove that entropy change during a reversible process is zero.
- (c) Verify TdS equation: $TdS = C_V dT + T \left(\frac{\partial P}{\partial T} \right)_V dV$
- (d) One mole of an ideal gas ($\gamma=1.4$), initially kept at 17°C , is adiabatically compressed so that its pressure becomes 10 times its original value. Calculate, its temperature after compression and work done on the gas.
- (e) Show that $K = \eta C_V$, where the symbols have their usual meanings.
- (f) Using Maxwell's law of distribution of molecular speed, derive expressions for most probable and root mean square velocities.
- (g) Describe experimentally the process of cooling due to adiabatic demagnetisation.
- (h) Give Einstein's theory of translational Brownian motion in gases.
- (i) Derive expression for coefficient of thermal conductivity of gases on the basis of kinetic theory of gases.
- (j) Derive expressions for Joule-Thomson coefficient for an ideal gas and a real gas.

Part IV

4. Answer any 4 of the following in 800 words (4x8)

- (a) What is Carnot's engine? Describe its operation with the help of a PV diagram and derive expression for its efficiency.
- (b) Define the four thermodynamic potentials. Using these potentials derive the four Maxwell's thermodynamic relationships.
- (c) Derive the Ehrenfest's equations for second order phase transitions.
- (d) Define mean free path of a gas molecule and derive its expression on the basis of kinetic theory of gases. Calculate the mean free path of molecules of a gas whose diameter is 2×10^{-8} cm and number density is $3 \times 10^{17} \text{ cm}^{-3}$.
- (e) Calculate the critical constants of Van der Waal's gas in terms of constants "a" and "b". Hence, derive the reduced equation of state.

DECEMBER -2025

WAVES AND OPTICS

Time- 3 hrs

Full marks-100

Answer as per instructions

Figures in the right-hand margin indicates marks.

Part – I

1. Objective Questions (All are compulsory)

10x1 = 10

- i) Fermat's principle states that light follows the path of
 - a) Maximum distance
 - b) Least optical path
 - c) Maximum optical path
 - d) Zero time
- ii) Which of the following is a cardinal point of an optical system?
 - a) Nodal point
 - b) Focal point
 - c) Principal point
 - d) All of the above
- iii) The Huygens eyepiece is free from
 - a) Chromatic aberration
 - b) Spherical aberration
 - c) Coma
 - d) Distortion
- iv) A wave is represented by $y=0.01\sin(200t-5x)$. What is its wavelength?
 - a) 0.628 m
 - b) 1.26 m
 - c) 0.2 m
 - d) 5 m
- v) Lissajous figures are obtained by the superposition of
 - a) Two parallel harmonic oscillations
 - b) Two perpendicular harmonic oscillations
 - c) Two damped oscillations
 - d) Two longitudinal waves
- vi) Interference due to division of wavefront occurs in
 - a) Michelson interferometer
 - b) Young's double slit experiment
 - c) Newton's rings
 - d) Fabry-Perot interferometer
- vii) In Young's double slit experiment, slit separation is 0.5 mm, wavelength is 600 nm, and screen distance is 1 m. Fringe width is
 - a) 0.6 mm
 - b) 1.2 mm
 - c) 0.3 mm
 - d) 2.4 mm
- viii) Newton's rings are used to determine
 - a) Wavelength of light
 - b) Refractive index
 - c) Radius of curvature
 - d) All of the above
- ix) The central maximum in single slit Fraunhofer diffraction is
 - a) Narrow
 - b) Bright and widest
 - c) Dark
 - d) Of zero width
- x) A zone plate produces multiple foci due to
 - a) Refraction
 - b) Reflection
 - c) Interference
 - d) Diffraction only

Part –II

2. Answer the following questions in 50 words

9x2= 18

- i) What is spatial coherence?
- ii) Light passes from air into glass of refractive index 1.6 at an angle of incidence 30° . Calculate the angle of refraction.
- iii) Explain energy transport in wave motion.
- iv) A wave has frequency 400 Hz and wavelength 0.75 m. Calculate the wave velocity.
- v) What is an air wedge and its application?
- vi) In a Michelson interferometer, the mirror is moved by 0.01 mm and 34 fringes are observed to shift. Find the wavelength of light used.
- vii) What is the principle of Fabry-Perot interferometer?
- viii) A diffraction grating has 4000 lines per cm. Calculate the grating element.
- ix) How does a zone plate differ from a convex lens?

Part – III

3. Answer any 8 out of 10 questions in 250 words

8x5 =40

- i) Describe the matrix formulation of geometrical optics and its advantages.
- ii) Explain cardinal points and cardinal planes of an optical system.
- iii) Compare Ramsden and Huygens eyepieces with neat ray diagrams.
- iv) Explain particle velocity and wave velocity in wave motion
- v) Two perpendicular SHMs of equal amplitudes have frequencies in the ratio 1:2. Sketch and identify the nature of the Lissajous figure formed.
- vi) Describe Lloyd's mirror experiment and explain the formation of fringes.
- vii) Explain the principle of Michelson interferometer. In a Michelson interferometer, the mirror is displaced by 0.02 mm and 60 fringes are observed to shift. Calculate the wavelength of light used.
- viii) Describe Lloyd's mirror experiment and explain the formation and characteristics of interference fringes.
- ix) Define resolving power of a telescope and derive its expression.
- x) Describe the theory of a zone plate and explain the formation of multiple foci.

Part –IV

4. Answer any 4 out of 5 Questions in 800 words

4x8= 32

- i) (a) Derive the lens formula for a thick lens using cardinal points. (6)
(b) Two thin lenses of focal lengths 10 cm and 20 cm are placed in contact. Find the equivalent focal length. (2)
- ii) (a) Derive the wave equation for a plane progressive wave and discuss its physical significance. (6)
(b) Show that the function: $y(x, t) = A e^{i(kx - \omega t)}$ is a wave. (2)
- iii) (a) Describe Newton's rings experiment and derive expressions for determining wavelength and refractive index. (6)
(b) In Newton's rings experiment, the radius of curvature of the lens is 1.5 m and wavelength of light is 589 nm. Calculate the radius of the 10th dark ring. (2)
- iv) (a) Describe diffraction due to a plane transmission grating and derive grating equation. (6)
(b) A plane transmission grating produces a first-order maximum at 30° for light of wavelength 600 nm. Calculate the grating spacing. (2)
- v) (a) Explain Fraunhofer diffraction at a single slit and derive the expression for intensity distribution. (6)
(b) For single-slit Fraunhofer diffraction, slit width is 0.1 mm and wavelength is 600 nm. Calculate the angular width of the central maximum. (2)
